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Part I: Oscillations and Mechanical Waves.

Any particle with a simple restoring force will result in simple harmonic motion:

$$F = -kx \Rightarrow x(t) = A\cos(\omega t + \phi)$$

$$\omega = \sqrt{\frac{k}{m}}$$

$\boxed{Q}$ How do you know?

$$F = ma = -kx \longrightarrow m\frac{d^2x(t)}{dt^2} = -kx(t)$$

try $x(t) = A\cos(\omega t + \phi)$

$$-mA\omega^2\cos(\omega t + \phi) = -kA\cos(\omega t + \phi)$$

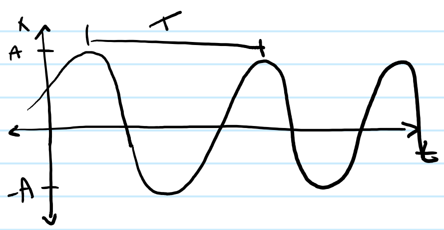
$$A\omega^2\cos(\omega t + \phi) = \frac{k}{m}A\cos(\omega t + \phi)$$

$$A\omega^2\cos(\omega t + \phi) = A\omega^2\cos(\omega t + \phi) \checkmark$$

This kind of force (and hence this kind of motion) is extremely common.

→ atoms in a solid
→ diatomic molecules
→ any elastic motion → (bridges, buildings)

ω is called the angular velocity of the particle. and ϕ is the phase constant.



The Measures of a wave:

(1) The period: time for a particle to come back to it's original position.

$$x(t) = x(t+T)$$

$$\Rightarrow \omega(t+T)+\phi - (\omega t+\phi) = 2\pi$$

$$\boxed{T = \frac{2\pi}{\omega}}$$

(2) The frequency : # of cycles per unit time.

$$T = \frac{\text{time}}{1\text{ cycle}} \Rightarrow f = \frac{1}{T}$$

Example:

$m_{car} = 1300\text{ kg}$ $\qquad k = 20\text{ kN/m}$

$m_{people} = 2\times 160\text{kg}$

4 springs.

Find the frequency of vibration after the car drives over a pothole in the road :

$$F = \Sigma(-kx) = -(\Sigma k)x$$

$k_{eff} = 80\ \frac{\text{kN}}{\text{m}}$

$m = 1460\text{ kg}$

$$\Rightarrow \boxed{f = \frac{1}{2\pi}\sqrt{\frac{k_{eff}}{m}} = 1.18\text{Hz}}$$

Energy of a Harmonic Oscillator

$$\boxed{E = \frac{1}{2}kA^2}$$

how do we know?

#1) at the peak amplitude $x = A$, there is no velocity so the energy is all potential

$$E = \frac{1}{2}kx^2 = \frac{1}{2}kA^2$$

#2) At the $x=0$ (equilibrium position), there is only Kinetic energy :

$$E = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{d}{dt}\left[A\cos(\omega t+\phi)\right]\right)^2$$

$$= \frac{1}{2}m\left(-A\omega\underbrace{\sin(\omega t+\phi)}_{\to 1 \text{ at } x=0}\right)^2$$

$$= \frac{1}{2}m\omega^2 A^2$$

$$= \frac{1}{2}kA^2$$

In general, $E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$

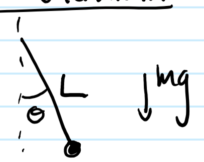
$$E = \frac{1}{2}m\omega^2A^2\sin^2(\omega t+\phi) + \frac{1}{2}kA^2\cos^2(\omega t+\phi)$$

$$= \frac{1}{2}kA^2\left(\sin^2(\omega t+\phi) + \cos^2(\omega t+\phi)\right)$$

$$\boxed{E = \frac{1}{2}kA^2}$$:)

Other examples of Simple Harmonic Motion:

The Pendulum:



$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta \Rightarrow \boxed{\omega = \sqrt{\frac{g}{L}}}$$

$$\boxed{T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}}$$

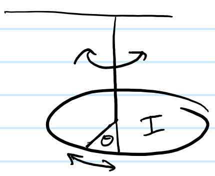
** the Physical Pendulum

$$\omega = \sqrt{\frac{mgd}{I}}$$

I = moment of inertia
m = total mass
d = distance from axis to CoM.

** Torsional Pendulum

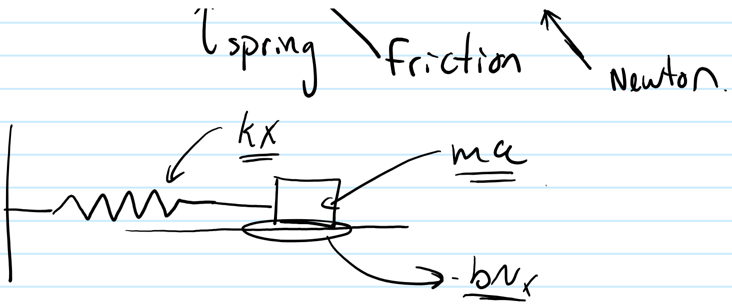
$$\omega = \sqrt{\frac{\kappa}{I}}, \quad T = 2\pi\sqrt{\frac{I}{\kappa}}$$



Damped Oscillations

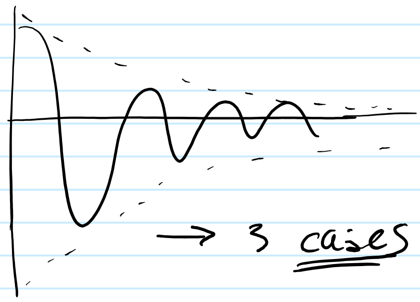
If there is a damping force as well as a restoring force the motion becomes "damped oscillation"

$$F = -kx - bv_x = ma_x$$



What is the final motion?

$$x(t) = A\, e^{\left(-\frac{b}{2m}t\right)} \cos(\omega t + \phi)$$



$$\omega = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$$

→ 3 cases

① if $\frac{k}{m} > \left(\frac{b}{2m}\right)^2$ → ω is real

* underdamped

② if $\frac{k}{m} < \left(\frac{b}{2m}\right)^2$ → ω is imaginary

⇒ * overdamped

③ $\frac{k}{m} = \left(\frac{b}{2m}\right)^2$ → critically damped

Forced Oscillation

If a particle is additionally driven with a force $F = F_0 \sin(\omega t)$, we find, again, simple harmonic motion.

$$x(t) = A\cos(\omega t + \phi)$$

Now, the amplitude is a function of the driving force:

$$\boxed{A = \frac{F_0/m}{\sqrt{(\omega^2 - \omega_0^2)^2 + \left(\frac{b\omega}{m}\right)^2}}}$$

Importantly, this amplitude is largest for a special ω.

$\boxed{\omega = \omega_0}$ this is

$\boxed{w = w_0}$ this is called $\boxed{\text{resonance}}$

Wave Motion

Two types:

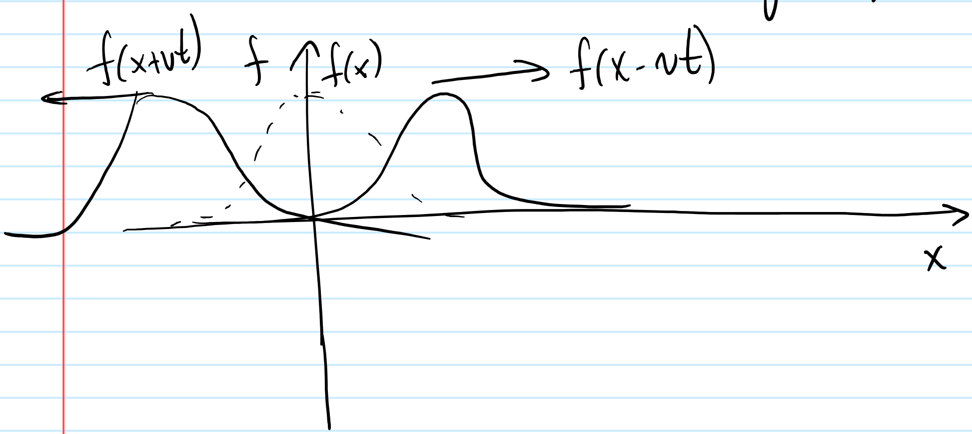
Transverse: disturbance perpendicular to dir. of travel
Ex: light, ocean wave

Longitudinal: disturbance parallel to dir. of travel
Ex: compressive wave, sound (sometimes),

Expressed as a function of $\vec{x}$ and t:

$f(x-vt)$ * forward moving (right)

$f(x+vt)$ * backwards moving (left)



Travelling Waves

$$f(x,t) = A\sin(kx - wt) \quad + \text{ phase}\ldots$$

all of the terminology:

v : velocity

v : velocity

k : angular wavenumber — ω : angular frequency

λ : wavelength — T : period

ν : wavenumber — f : frequency

The key idea is that the spatial parts and temporal parts are linked by $\boxed{v = \frac{\lambda}{T}}$

Waves on Strings

the velocity of waves on strings : $v = \sqrt{\frac{T}{\mu}}$

μ = mass density
T = tension density

reflections

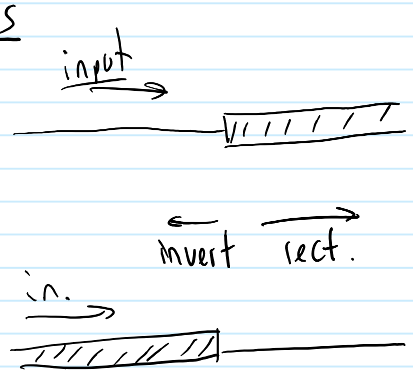
↳ at a hard boundary, the pulse on a string is inverted on reflection.

↳ at an "open boundary", the pulse on a string is not inverted

→ the same is true for light

Transmissions

↳



$\overleftarrow{rect}$ rect

Power transmitted

$$\boxed{P = \frac{1}{2}\mu\omega^2 A^2 v}$$

Compare with one oscillator $E = \frac{1}{2}m\omega^2 A^2$ /or

The Wave Equation

$$\boxed{\nabla^2 f = \frac{1}{v^2}\partial_t^2 f}$$

Sound Waves

Example of a longitutinal wave in pressure.

$$\boxed{\Delta P(x,t) = \Delta P_{max}\sin(kx-\omega t+\phi)}$$

The velocity v is $v = \sqrt{\frac{B}{\rho}}$ B = bulk modulus

ρ = volume density

Wave Intensity

$$I \equiv \frac{P_{avg}}{A} \longrightarrow I = \frac{P_{avg}}{4\pi r^2}$$

spherical rays

Decibels

β, the sound level is quantity that descibes the loudness on a logarithm scale:

$$\boxed{\beta = 10\log\left(\frac{I}{I_0}\right)}$$

Why a log scale? Human perception seems to work multiplicatively instead of additively.

I_0 is the threshold of hearing.

$$\boxed{I_0 = 1.00 \times 10^{-12}\ W/m^2}$$

Doppler Shift

$$f' = \left(\frac{v}{v - v_s}\right) f$$ *(moving towards observer)

$$f' = \left(\frac{v}{v + v_s}\right) f$$ *(moving away from observer)

generally $$\boxed{f' = \left(\frac{v + v_o}{v - v_s}\right) f}$$

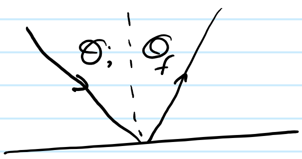
v_o = observer velocity
v_s = source velocity

Part II - Light and Optics

Ray Optics

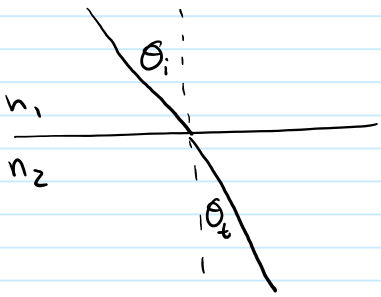
Law of reflection:

$$\boxed{\theta_i = \theta_r}$$



Snell's Law

$$\boxed{n_1 \sin\theta_i = n_2 \sin\theta_2}$$



n= index of refraction

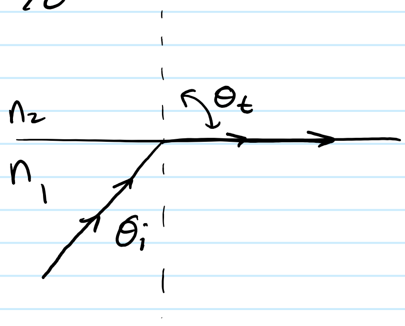
$$\boxed{n = \frac{c}{v}}$$

In general, 'n' is function of the wavelength, leading to a phenomena called 'dispersion'.

Total internal reflection

Light travelling from high to low index media can be completely reflected back into the media. this happens when $\theta_t = 90^\circ$

eg

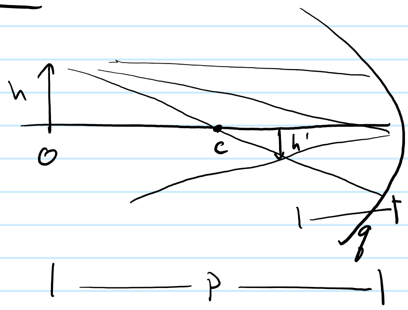


$$\boxed{\sin\theta_c = \frac{n_2}{n_1}}$$

$$\boxed{n_1 > n_2}$$

Imaging

Mirrors



$$\Longrightarrow \boxed{M = -\frac{q}{p}}$$

mirror equation

$$\boxed{\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}} \qquad \boxed{f = \frac{R}{2}}$$

Ray Diagrams

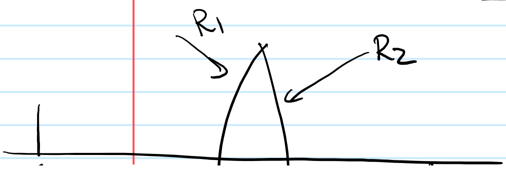
(1) Parallel ray $\rightarrow$ through focus
(2) Ray through ray $\rightarrow$ out parallel
(3) Ray through center $\rightarrow$ straight

center of curvature for mirror
center of lens for lens.

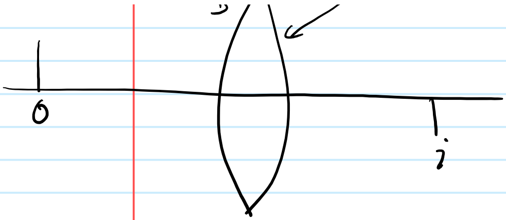
Lenses

$$\boxed{\frac{n_1}{s_o} + \frac{n_2}{s_i} = \frac{n_2 - n_1}{R}}$$

$\Longrightarrow$ lens Maker Equation



$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$



$$\boxed{\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f} \qquad f = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

Adding lenses

$\rightarrow \boxed{\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2}}$

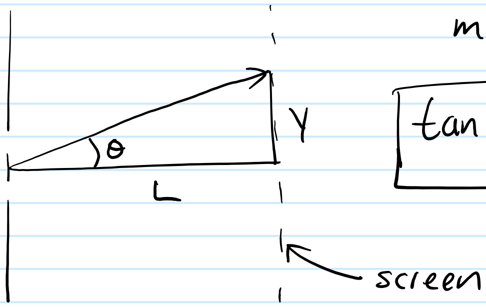
Wave Optics

Young's Double Slit

$\boxed{\underbrace{d \sin\theta_{bright}}_{\delta} = m\lambda}$ $\qquad$ $\boxed{d \sin\theta_{dark} = (m+\tfrac{1}{2})\lambda}$

$\delta \rightarrow$ path difference

$m \in \mathbb{Z}$



$\boxed{\tan\theta = \frac{Y}{L}}$

Intensity $\boxed{I = I_{max} \cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right)}$

Thin film interference

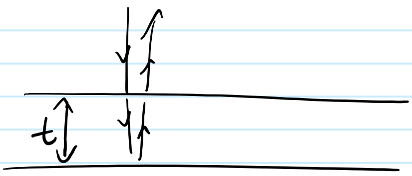
Constructive

$\boxed{2t = (m+\tfrac{1}{2})\lambda_n}$ $\qquad m = 0, 1, 2 \ldots$

Destructive

$\boxed{2t = m\lambda_n}$ $\qquad m = 0, 1, 2 \ldots$

$\lambda_n = \lambda n$

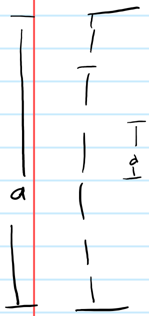


Diffraction Patterns and Polarization

Diffraction Grating

$\boxed{a \sin \theta_{dark} = m\lambda}$ $m \in \mathbb{Z}$ from a single slit

↳ envelope diffraction



$\boxed{d \sin \theta_{bright} = m\lambda}$ → interference (diffraction grating)

Part III – "Modern Physics"

Relativity

$$\boxed{\begin{gathered} L(v) = L(0)\sqrt{1 - v^2/c^2} \\ T' = T\left(1 - \frac{v}{c}\right) \end{gathered}}$$

→ (1) physics does not depend on velocity of frame

→ (2) the speed of light is always "c".

$\boxed{E = mc^2}$ at rest

Quantum Mechanics

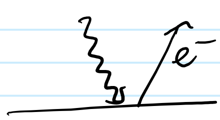
Photons : $E = hf = \hbar\omega$

energy of photon

$E = pc = hf$

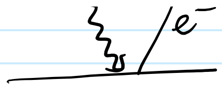
$\boxed{p = \frac{h}{\lambda}}$ ⟶ deBroglie $\boxed{\lambda = \frac{h}{p}}$

Photo Electric



$\boxed{\frac{1}{2} m_e v^2 = hf - W}$

work function



$$\boxed{\frac{1}{2} m_e v = hf - W}$$

Work function

Blackbody Radiation

$$\boxed{u(f,T) = \frac{8\pi f^2}{c^3} \frac{hf}{\exp\{hf/k_BT\} - 1}}$$

Bohr Atom

$$\boxed{E_n = -\frac{\hbar^2}{2ma_0^2} \frac{Z^2}{n^2}}$$

$$\boxed{a_0 = \frac{\hbar}{mc\alpha}}$$

$$\boxed{R_n = \frac{n^2}{Z} a_0}$$

$$\boxed{\alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c}}$$

$$\boxed{E_n = -\frac{1}{2} m \frac{(Z\alpha c)^2}{n^2}}$$

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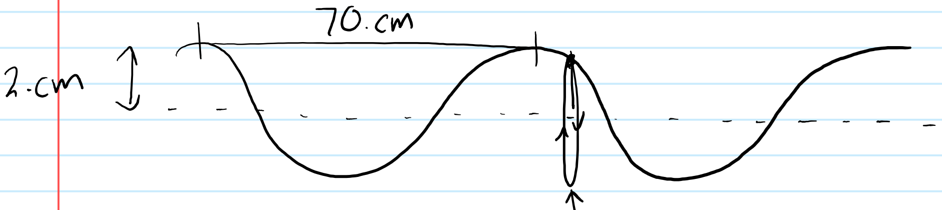
Examples

Dawson #1



$f = \dfrac{200 \text{ cycles}}{12.0\,s}$

a) $V = ?$ $\quad V = \dfrac{T}{\lambda}$ $\quad T = \dfrac{12.0\,s}{200 \text{ cycles}}$ $\quad \lambda = 70.0\,cm$

$$\Rightarrow \boxed{V = 8.571\ \frac{cm}{s}}$$

What is the maximum transverse speed?

at a point x

$$x(t) = A\cos(wt + \phi)$$

what is w?

$$T = \frac{2\pi}{w} \Rightarrow w = \frac{2\pi}{f}$$

$$\boxed{w = 0.377\ \frac{rad}{s}}$$

$$v(t) = -Aw\sin(wt+\phi)$$

$$V_{max} = Aw = 2.0\,cm * 0.377\ \frac{rad}{s}$$

$$\boxed{V_{max} = 0.754\ \frac{cm}{s}}$$

b) Explain

c) $w = \sqrt{\frac{T}{\ldots}} \longrightarrow \mu = \frac{T}{\ldots} =$ answer.

b) Explain

c) $w = \sqrt{\frac{T}{\mu}} \longrightarrow \mu = \frac{T}{w^2} = \underline{\text{answer}}.$

d) $P = \frac{1}{2}\mu w^2 A^2 v$

Multiple Choice